Qubit mediated, time robust entangling of oscillators in thermal environments

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Outline

- Standard Qubit-mediated entanglement preparation
- Qubit-oscillator interactions
- Entanglement detection
- Conclusions

Entanglement: a fundamental resource for QIP

Possible definition of Entanglement

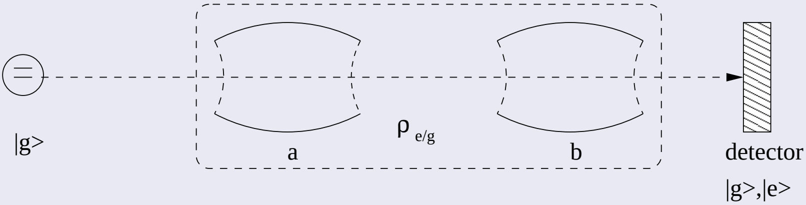
correlations between subsystems of a multipartite system that cannot be created by Local Operations + Classical Communication (LOCC). $\Leftrightarrow$ non separability

Maximally entangled 2-Qubit states: Bell States

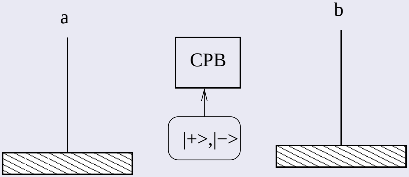
$$|\psi^{\pm}\rangle = \frac{1}{\sqrt{2}}\left(|ge\rangle \pm |eg\rangle\right), \qquad |\phi^{\pm}\rangle = \frac{1}{\sqrt{2}}\left(|gg\rangle \pm |ee\rangle\right). \tag{1}$$

Entanglement preparation via Qubit-oscillator interaction

preparation - 1



preparation - 2



Jaynes Cummings interaction

Qubit $\rightarrow \{|g\rangle, |e\rangle, \sigma_i, \sigma^{\pm}\}$, Oscillator $\rightarrow \{a, a^{\dagger}, |n\rangle\}$, we assume **resonance condition** and consider the **interaction picture**

JC Hamiltonian

$$H_{\mathrm{JC}} = (a\sigma^{+} + a^{\dagger}\sigma^{-})$$

$$E_n^{\pm} = \pm\sqrt{n}, \qquad |e_n^{\pm}\rangle = \frac{1}{\sqrt{2}}(|g, n\rangle \pm |e, n-1\rangle)$$

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- we like: simple and intuitive
- we **don't** like: irrational frequency ratios!

$\Rightarrow$ the evolution of the components $c_n = \langle n|\psi\rangle$ will not relate in a simple way.
Fine tuning of the interaction time is usually needed in the existing entanglement preparation protocols

Position Coupling

$$H = \sigma_1(a + a^\dagger)$$

Time evolutor

$$U(t) = e^{-iHt} = D(-i\sigma_1 t)$$

$$D(\alpha) = e^{\alpha a^\dagger - \alpha^* a}$$

Easily generalized for 2+ oscillators:

$$U_2(t) = D_a(-i\sigma_1 t) D_b(-i\sigma_1 t),$$

If the Qubit is in g at time 0, then measured in g at time t:

$$\Pi_g = (U_2(t))_{g,g} = \frac{1}{2}(D_a(it)D_b(it) + D_a(-it)D_b(-it)).$$

Entanglement due to combination of displacements
$\Rightarrow$ robustness to δt, $T > 0$

Master equation

$T > 0$ must be taken into account! For a single oscillator:

Master equation

$$\partial_t \rho = -i[\sigma_1(a + a^\dagger), \rho] + \kappa L(T)\rho;$$

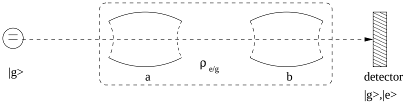
Analytical solution via the Wigner Representation:

$$W(\alpha) = \frac{1}{\pi^2} \int d^2\eta \;\; e^{\eta^* \alpha - \eta \alpha^*} \mathrm{Tr}\{D(\eta)\rho\},$$

It's a "mixed" representation, W is a **matrix**.
For 1 qubit and 2 oscillators: 2x2 matrix, 2 phase space variables

$$\begin{aligned} & W_{ij}(\alpha, \beta); \\ & i, j = \mathrm{e,g} \end{aligned} \qquad (2)$$

Entangling thermal oscillators - Bell-CHSH inequality



Qubit found in g after flying time t, temperature $T \Rightarrow$ state of the oscillators described by

$$W_g(\alpha, \beta, t, T) = (\mathcal{N}(t, T))^{-1}(W(\alpha, \beta, t, T))_{g,g} \tag{3}$$

To verify nonlocal properties of the oscillators we consider a **Bell-CHSH inequality test**:

$$\mathcal{B} = \left| \frac{\pi^2}{4} \left(W_g(\alpha, \beta) + W_g(\alpha', \beta) + W_g(\alpha, \beta') - W_g(\alpha', \beta') \right) \right|$$

LOCC

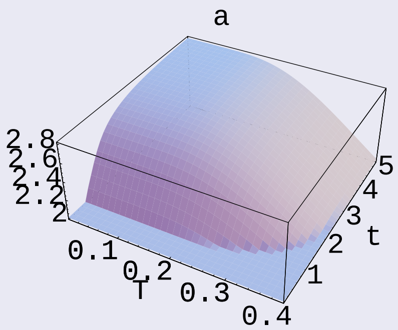
$$\mathcal{B} \leq 2$$

NON LOCC

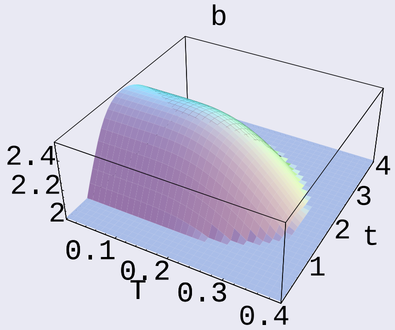
$$2 \leq \mathcal{B} \leq 2\sqrt{2}$$

Entangling thermal oscillators - Bell-CHSH violation

$\kappa = 0$

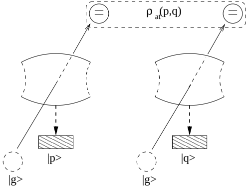


$\kappa = 0.01$



- No need of fine tuning in time!
- Entanglement verifiable for temperatures up to $T_c \simeq 0.408\omega$
- Decoherence and thermalization reduce temperature and time windows

Entanglement Reciprocation



Entanglement is present also for $T > T_c$, which can be verified *conditionally* with probe Qubits:

After the interaction time t we measure the observables

$$\hat{p}_a = -i(a - a^\dagger); \hat{p}_b = -i(b - b^\dagger)$$

Output Qubit state

$$\rho_{p_a,p_b}(t,t) = \frac{\langle p_a, p_b|\rho(t,t)|p_a, p_b\rangle}{\mathrm{Tr}\{\langle p_a, p_b|\rho(t,t)|p_a, p_b\rangle\}},$$

$$\langle p_a, p_b|\rho(t,t)|p_a, p_b\rangle = \int dxdyW(x + ip, y + iq, t, t)$$

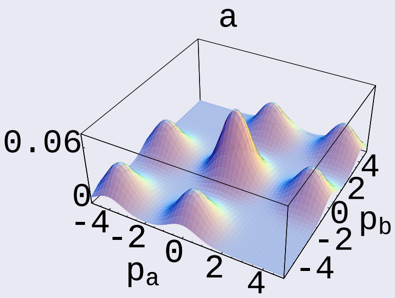
No additional entanglement created at this stage!

Qubit Entanglement

Snapshot for $\kappa = 0.01, t = 2$ and $T = \omega$.

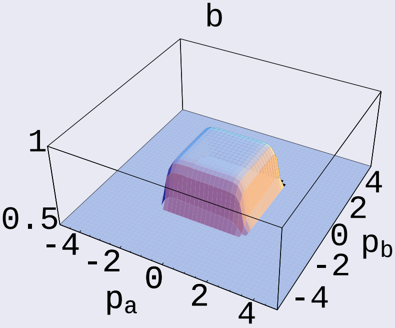
Probability of the outcome (p_a, p_b)

$$P(p_a, p_b) = \mathrm{Tr}\{\langle p_a, p_b|\rho|p_a, p_b\rangle\}$$



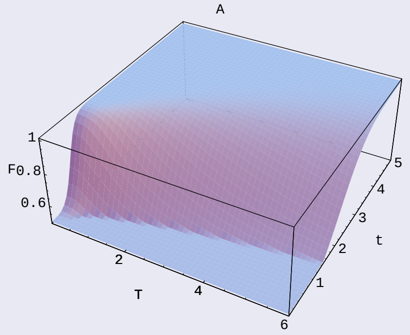
Maximally entangled fraction of ρ_{p_a,p_b} ("Fidelity")

$$F = \langle\psi^+|\rho_{p_a,p_b}|\psi^+\rangle$$

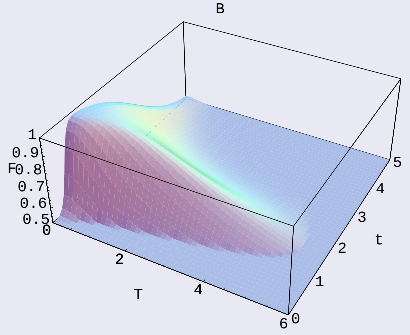


Qubit Entanglement / 2

Max Fidelity, $\kappa = 0$



Max fidelity, $\kappa = 0.01$



- Wide range of times and temperatures. No fine tuning
- Ideally no temperature limit for $\kappa \to 0$
- Max Fidelity corresponds to $P_{\max} \simeq 25\%$ for low T

Conclusions

- Considering a different interaction Hamiltonian allowed more flexibility in the typical entanglement preparation protocols involving Qubits and harmonic oscillators
- Nonzero temperature decoherence can be taken into account analytically via Wigner Representation
- Entanglement between the oscillators can be proven via the Bell-CHSH test for $T < T_c$
- Entanglement can be reciprocated to Qubit pairs even for $T > T_c$, conditional to the measurement outcomes of the momenta of the oscillators
- No need of fine tuning in time, the allowed time window becoming larger for smaller κ and T